Abstract

Vector data is prevalent across business and scientific applications, and its popularity is growing with the proliferation of learned embeddings. Vector data collections often reach billions of vectors with thousands of dimensions, thus, increasing the complexity of their analysis. Vector search is the backbone of many critical analytical tasks, and graph-based methods have become the best choice for analytical tasks that do not require guarantees on the quality of the answers. Although several paradigms (seed selection, incremental insertion, neighborhood propagation, neighborhood diversification, and divide-and-conquer) have been employed to design in-memory graph-based vector search algorithms, a systematic comparison of the key algorithmic advances is still missing. We conduct an exhaustive experimental evaluation of twelve state-of-the-art methods on seven real data collections, with sizes up to 1 billion vectors. We share key insights about the strengths and limitations of these methods; e.g., the best approaches are typically based on incremental insertion and neighborhood diversification, and the choice of the base graph can hurt scalability. Finally, we discuss open research directions, such as the importance of devising more sophisticated data adaptive seed selection and diversification strategies. This work was presented at the ICML 2025 VecDB Workshop. An extended version appeared in ACM SIGMOD 2025.

1. Introduction

Vector data is common in various scientific and business domains, and its prevalence is expected to grow in the future with the proliferation of learned embeddings. The volume and dimensionality of this data, which can exceed multiple terabytes and thousands of dimensions, make its analysis very challenging. A critical component of these data analysis tasks is vector search (Echihabi et al., 2021; Palpanas, 2015). It supports recommendation (Wang et al., 2018), information retrieval (Williams et al., 2014), clustering (Bubeck & von Luxburg, 2009), classification (Petitjean et al., 2014), entity resolution (Christophides et al., 2020), outlier detection (Boniol & Palpanas, 2020) and AI explainability (Kaneko, 2023) in many fields including bioinformatics, computer vision, finance. More recently, vector search has been playing a crucial role in improving the performance and interpretability of Large Language Models and reducing their hallucinations (Blattmann et al., 2022; Karpukhin et al., 2020).

As these applications scale to ever-larger datasets, often terabytes in size and thousands of dimensions, vector search becomes not only a computational bottleneck, but also a significant contributor to system latency and energy consumption (Huang et al., 2024). In many AI pipelines, especially those deployed at inference time, similarity search dominates runtime cost (Desislavov et al., 2023). Optimizing vector search is therefore essential to building applications that are not only faster and more responsive, but also more efficient in terms of operational resources and environmental impact (Chiang et al., 2025).

A vector search algorithm over a dataset S of n d-dimensional vectors returns answers in S that are similar to a given input vector V_Q. The brute-force approach (a.k.a. a sequential or serial scan) compares V_Q to every single element in S, with time complexity $O\,(nd)$—which becomes impractical and energy-intensive for large, high-dimensional datasets. To address this, state-of-the-art methods reduce the dimensionality d via compact representations and/or minimize the number of comparisons n through efficient indexing and pruning strategies. These improvements not only accelerate search but also reduce memory usage and computational overhead, contributing to more energy-efficient systems. Some approaches compute exact answers, while others ϵ- and δ-ϵ-approximate answers, with deterministic and probabilistic guarantees on the accuracy of the answers, or ng-approximate answers without any theoretical guarantees, but high accuracy in practice (Echihabi et al., 2019).

A large body of work has been dedicated to approximate vector search, which trades off accuracy for efficiency. These approaches are based on scans (Weber et al., 1998; Ferhatosmanoglu et al., 2000), trees (Camerra et al., 2014; Wang et al., 2025), graphs (Malkov & Yashunin, 2020; Subramanya et al., 2019; Munoz et al., 2019), inverted indexes (Jégou et al., 2011; Babenko & Lempitsky, 2015), hashing (Sun et al., 2014a;b), or a combination of these data structures (Chen et al., 2018; Azizi et al., 2023). Over the last decade, graph-based techniques have emerged as the method of choice for vector search in many real applications that can relax theoretical guarantees to achieve a query latency of a few milliseconds on terabyte-scale collections (Song et al., 2020; Johnson et al., 2019).

Graph-based methods typically represent a dataset S as a proximity graph $G(V, E)$, where V is the set of vertices corresponding to the n data points, and E is the set of edges connecting similar points. Query answering for a given query node V_Q usually involves running an ng-approximate beam search, starting with an initial set of seed nodes to warm up the candidate priority queue, followed by a greedy graph traversal expanding promising candidates. State-of-the-art (SotA) graph-based methods share this beam-search approach but differ mainly in their graph construction strategies and seed selection mechanisms.

Although several paradigms (seed selection, incremental insertion, neighborhood propagation, neighborhood diversification, and divide-and-conquer) have been employed to design in-memory graph-based vector search algorithms, a systematic comparison of the key algorithmic advances is still missing. The only empirical study dedicated entirely to this family of techniques is (Wang et al., 2021) without providing conclusive results due to the small scale of datasets used in experiments, not exceeding 1M vectors. As we will show in Section 3 that trends change as dataset size increases. Moreover, existing benchmarks (Aumüller et al., 2017; Simhadri et al., 2022) for evaluating vector search methods do not focus on graph-based approaches, and thus, do not shed light on why the best methods have superior query performance. To the best of our knowledge, our work is the first proposing a taxonomy of algorithmic advances grounded in the actual indexing and search design principles, rather than on abstract graph categories. This enables new insights into how key design choices impact the indexing and search performance across varying dataset sizes, addressing limitations in prior studies.

In this paper, we make the following contributions[1]:

• We identify five core paradigms underlying graph-based vector search: Seed Selection (SS), Neighborhood Propagation (NP), Incremental Insertion (II), Neighborhood Diversification (ND), and Divide-and-Conquer (DC). We briefly overview these paradigms and focus deeply on SS and ND, due to their significant impact on performance.

• We propose a taxonomy categorizing existing methods according to these paradigms, highlighting their chronological evolution and influence.

• We briefly survey state-of-the-art methods, discussing their design principles, strengths, and limitations.

• We conduct an extensive evaluation of twelve state-of-the-art methods on synthetic and real-world datasets (up to 1 billion vectors). Our experiments validate conventional wisdom, such as the superiority of incremental insertion methods in query performance and scalability (Azizi et al., 2023; Wang et al., 2021). However, we also find differences from recent studies (Wang et al., 2021), notably demonstrating better-than-previously-reported efficiency of SPTAG-BKT (Chen et al., 2018) on small datasets and superior indexing efficiency of HNSW (Malkov & Yashunin, 2020). Contrary to prior findings (Azizi et al., 2023), we also show that Vamana (Subramanya et al., 2019) is competitive on large scale datasets.

• We provide novel insights, such as (i) identifying the most effective ND technique for large datasets, and (ii) analyzing the impact of different SS methods on query and indexing performance on various dataset scales.

• Finally, we highlight promising research directions, including developing scalable NP and ND graph structures, data-adaptive seed selection techniques, theoretical analysis of ND methods, and tailored strategies (e.g., clustering, diversification) for DC-based approaches.

2. Graph-Based Vector Search

We now present an overview of the main SotA graph-based ng-approximate vector search methods. We outline the base data structures and algorithms in this field, and identify five main paradigms exploited by the SotA approaches. We propose a new taxonomy that categorizes these approaches along the five paradigms, highlighting also their chronological development and influence map.

2.1. A Primer

A proximity graph is a graph $G(V, E)$ in which two vertices V_i and V_j are connected by an edge if and only if they satisfy particular geometric requirements, namely the *neighborhood criterion* (Shamos & Hoey, 1975). A proximity graph can be constructed using different distances such as the dot product (Morozov & Babenko, 2018), nevertheless the Euclidean distance remains the most popular one (Edelsbrunner, 2012). One of the earliest proximity graphs in the literature is the Delaunay Graph (*DG*). It is a planar dual

[1]Full paper appeared as (Azizi et al., 2025).

Algorithm 1 Beam Search (G, q, s, k, l)

Input: Graph G, query vector q, initial seeds s, result size k, beam width $l \geq k$
Output: k approximate nearest neighbors to q
Initialize candidate set $C \leftarrow s$
Initialize visited list $V \leftarrow \emptyset$
while $C \setminus V \neq \emptyset$ **do**
  $p^* \leftarrow \text{argmin}_{x_i \in C \setminus V}\, dist(q, x_i)$
  $C \leftarrow C \cup N_{out}(p^*)$
  $V \leftarrow V \cup \{p^*\}$
  if $|C| > l$ **then**
    Retain only the closest L points in C to q
  end if
end while
Return the closest k candidates in C to q

graph for the Voronoi Diagram (Fortune, 1995), where each vertex is the center of its own voronoi cell, and two vertices are linked if and only if their corresponding voronoi cells share at least one edge. A DG satisfies the Delaunay Triangulation :$\forall q, p, r \in V, (q,p), (q,r), (r,p) \in E$ if the circumcircle of the triangle q, p, r is empty (Aurenhammer et al., 2013). A beam search (Reddy et al., 1977) (Algorithm 1) on a *DG* can find the exact nearest neighbors (Dobkin et al., 1990) when the dataset has a high dimensionality or the beam search uses a large beam width. However, using a DG in high dimensions is impractical, because the graph becomes almost fully connected as the dimensionality d grows (Dobkin et al., 1990). Thus, SotA methods build alternative graph structures and use beam search to support efficient query answering (Gabriel & Sokal, 1969; Matula & Sokal, 1980; Toussaint, 2002).

2.2. Main Paradigms

We provide a brief overview of the five main paradigms exploited by SotA methods. Then, we describe in more detail the two paradigms that have the greatest impact on query performance (as will be demonstrated in Section 3).

Seed Selection (SS) chooses initial nodes to visit during search. It is also used during index building by approaches that exploit a beam search during the construction of the index to decide which edges to build. Some methods simply select one or more seed(s) randomly, while others use special data structures, e.g., a K-D Tree.

Neighborhood Propagation (NP) refines a pre-existing graph following a user-defined number of iterations, a.k.a. NNDescent (Dong et al., 2011). During each iteration, the potential neighbors of a given node are sourced both from its immediate neighbors and the neighbors of its neighbors. Then, the node only keeps the m closest neighbors, where m is a user-parameter. The pre-existing graph could be a random graph or some other type of graph.

Incremental Insertion (II) refers to building a graph by inserting one vertex at a time. Each vertex is connected using bi-directional edges to its nearest neighbors and some distant vertices. The neighbors are selected using a beam search on the already inserted portion of the graph. At the end of graph construction, some vertices retain early connections which act as long-range links. This approach was first proposed in the object-based peer-to-peer overlay network VoroNet (Beaumont et al., 2007a), with the idea of adding long-range links being inspired from Kleinberg's small-world model (Kleinberg, 2000; Kleinberg et al., 2002), with the difference that the latter selects the long-range links randomly.

Neighborhood Diversification (ND) was first introduced by the Relative Neighborhood Graph (RNG) (Toussaint, 1980). It aims to sparsify the graph by pruning unnecessary edges while preserving connectivity. For each node, ND exploits the geometrical properties of the graph to remove edges to neighbors that lead to redundant regions or directions. This indirectly causes the creation of long-range links allowing nodes to maintain diversified neighborhood lists, which reduces the number of comparisons during search.

Divide-and-Conquer (DC) is a strategy that splits a dataset into multiple, possibly overlapping, partitions, then builds a separate graph on each partition. Some approaches such as SPTAG (Chen et al., 2018) and HCNNG (Munoz et al., 2019) combine the individual graphs into one large graph, on which a beam search is performed, while ELPIS (Azizi et al., 2023) maintains the graphs separate and searches them in parallel.

2.3. Seed Selection

While state-of-the-art graph-based vector search methods vary in graph construction strategies, nearly all rely on beam search (Algorithm 1) for query answering, as it efficiently retrieves good answers in well-connected graphs. However, the choice of initial nodes—referred to as *seeds*—significantly impacts search efficiency: better seeds lead to fewer visited nodes and faster searches. Typically, an entry node or multiple candidate entry nodes are used to warm the beam search's priority queue; in the case of multiple seeds, the search picks the one closest to the query as the initial node, retaining others in the priority queue. While several methods propose additional indexes built on data samples to facilitate seed selection, none provide comprehensive evidence supporting their effectiveness. In this paper, we systematically examine the seed-selection techniques proposed in the literature:

(1) **Stacked-NSW (SN)**: HNSW (Malkov & Yashunin, 2020), inspired by skip lists (Pugh, 1990), builds hierar-

chical NSW graphs by sampling nodes from lower layers. Nodes are assigned a top layer $L = -\ln(\xi)/\ln(M/2)$, with $\xi \sim U(0,1)$ and out-degree M. Queries descend greedily from a fixed top-layer entry point, returning the closest sampled node to the query as entry point.

(2) **K-D Trees (KD)**: Constructs a single or multiple K-D Tree(s) (Beis & Lowe, 1997) on a dataset sample. During search,retrieves the set of seed points is retrieved through depth-first search traversal on the K-D Tree structure(s).

(3) **LSH**: Constructs an LSH index on a sample of the dataset to retrieve the seeds during search.

(4) **Medoid (MD)**: Uses medoid node as seed and entry point during query answering.

(5) **Single Fixed Random Entry Point (SF)**: A random node is selected and fixed as the entry point for all searches.

(6) **K-Sampled Random Seeds (KS)**: For each query, k random nodes are selected as seed points.

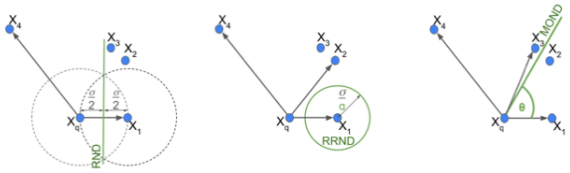
(7) **Balanced K-means Trees (KM)**: Constructs Balanced K-means Trees (BKT) (Malinen & Fränti, 2014) on a dataset sample. During search, seed points are retrieved via depth-first search on the BKT structure(s).

2.4. Neighborhood Diversification

Neighborhood Diversification (ND) builds sparse graphs by selectively pruning edges to balance short and long-range connections, thus reducing redundant comparisons. Initially proposed by RNG (Toussaint, 1980; 2002), ND was adapted for approximate vector search by several graph-based methods. Three main ND strategies exist: Relative Neighborhood Diversification (RND) used by HNSW (Malkov & Yashunin, 2020), NSG (Fu et al., 2019), SPTAG (Chen et al., 2018) and ELPIS (Azizi et al., 2023); Relaxed RND (RRND) introduced by Vamana (Subramanya et al., 2019); and Maximum-Oriented ND (MOND), proposed by DPG (Li et al., 2019) and NSSG (Fu et al., 2021). RND connects node X_q to candidate X_j only if no current neighbor X_i is closer to X_j. RRND relaxes this condition by factor $\alpha \geq 1$, reducing pruning and increasing edges. MOND selects edges based on an angle threshold $\theta \geq 60°$, favoring diverse edge orientations. RRND and MOND prune fewer edges than RND, thus resulting in denser graphs (cf. Figure 1). Note that any nodes pruned by RRND and MOND will eventually be pruned by RND, but not vice versa. Refer to (url, 2025) for a detailed proof.

2.5. A Taxonomy

Figure 2 depicts the SotA graph-based approaches, classified based on the five design paradigms: SS, NP, II, ND, and DC. The taxonomy also reflects the chronological development of the methods. Directed arrows indicate the influence of one method on another. Within the ND category, distinctions are made between different strategies, i.e., No Neighborhood Diversification (NoND), RND, RRND, and MOND (cf. Section 2.4). We identify the SS strategy of each method: KS, KD, SN, MD, LSH, and KM (SF is not used by any SotA method, but we consider it as an alternative strategy).



(a) RND (b) RRND (c) MOND

Figure 1. Neighborhood diversification approaches

Additionally, some methods use more than one strategy (e.g. NSG and VAMANA use KS and MD), or offer the flexibility to use different strategies (e.g., SPTAG can use either KD or KM). Note that a method can exploit one or more paradigms; e.g., HNSW uses incremental node insertion and prunes each node's neighbors using the RND approach, thereby being classified as both II and ND. KGraph (Dong, 2022) was the first to use NP to approximate the exact k-NN graph (k-NNG) (with quadratic complexity), and influenced numerous subsequent methods, including IEH (Jin et al., 2014) and EFANNA (Fu & Cai, 2016). In parallel, NSW (Ponomarenko et al., 2011) introduced the II strategy for graph construction.

HNSW (Malkov & Yashunin, 2020) and DPG (Li et al., 2019) leveraged ND to enhance NSW and KGraph, respectively. The good performance of HNSW and DPG encouraged more methods to adopt the ND paradigm, including NGT (Yahoo Japan Corporation, 2022), NSG (Fu et al., 2019) and SSG (Fu et al., 2021), which apply ND on the NP-based graph EFANNA. SPTAG (Chen et al., 2018) combined DC with ND.

Vamana (Subramanya et al., 2019) adopts NSG's idea of constructing the graph through beam search and ND. However, Vamana constructs its graph by refining an initial base random graph in two rounds of pruning, using RRND and RND. Inspired by HNSW, Vamana and NGT proposed variants that support incremental graph building (Croft et al., 2021; Yahoo Japan Corporation, 2022), but we classify them as ND-based per the ideas proposed in the original papers. HCNNG (Munoz et al., 2019) was influenced by SPTAG and adopted a DC approach for constructing the graph without adopting ND. ELPIS (Azizi et al., 2023) also adopted a DC strategy but leveraged both II and ND. HVS (Lu et al., 2021) and LSHAPG (Zhao et al., 2023) both propose new seed selection structures for HNSW, with the latter addition-

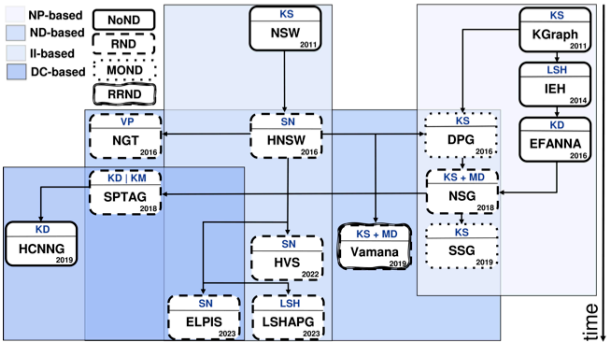


Figure 2. Graph-based ANN indexing paradigms

ally adopting a new probabilistic rooting approach. Note that earlier approaches, except from NSW, were mainly NP-based; however, recent studies have focused on devising methods that leverage the ND, II, and DC paradigms because they lead to superior performance (cf. Section 3).

3. Experimental Evaluation

4. Discussion

5. Conclusions

In this paper, we conduct a survey of the SotA graph-based methods for in-memory ng-approximate vector search, proposing a new taxonomy based on five key design paradigms. Through extensive experimentation on datasets with up to 1B vectors, we highlight the scalability challenges faced by most methods, with incremental insertion methods showing the best scalability on datasets exceeding 100GB. We observe that light-weight hierarchical structures help select better seeds to start the search on billion-scale datasets, and that neighborhood diversification is a key contributor in improving the query answering performance, with RND and MOND being the best techniques. We also propose promising research directions.

Acknowledgements

Impact Statement

Paper presents work that advances field of Machine Learning; many potential societal consequences, none of which must be specifically highlighted here.

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A. State-of-the-Art Approaches

This section presents the graph-based approximate nearest neighbor (ANN) methods studied in our experiments, highlighting their core principles:

KGraph (Dong, 2022) reduces the construction cost of an exact k-NNG, which has a quadratic worst-case complexity. It constructs an approximate k-NNG by refining a random initial graph with an empirical cost of $O\left(n^{1.14}\right)$ (Dong et al., 2011). This refinement process, also known as NNDescent (Dong et al., 2011) (Neighborhood Propagation), aims at improving the approximation of the k-NN graph by assuming that the neighbors of a vertex u are more likely to be neighbors of each other. The process iterates over all graph vertices $u \in V$: for each vertex u and pair (x, y) of its neighbors, it adds x to the neighbors of y and vice-versa, keeping the closest k neighbors of u.

Navigable Small World (NSW) (Ponomarenko et al., 2011; Malkov et al., 2014) is an approximation of a Delaunay graph which guarantees the small world property (Watts & Strogatz, 1998), i.e. the number of hops L between two randomly chosen vertices grows to the logarithm of graph size n such that $L \propto Log\,(n)$. An *NSW* graph is based on the VoroNet graph (Beaumont et al., 2007a), an extention of Kleinberg's variant of Watts-Strogatz's small world model graph (Kleinberg, 2000; Kleinberg et al., 2002), The VoroNet graph is built incrementally by inserting a randomly picked vertex to the graph and connecting it to 2d+1 neighbors selected using a beam search on the existing vertices in the graph. Once this process completes, the first built edges would serve as long-range edges to quickly converge toward nearest neighbors (Beaumont et al., 2007a). The resulting graph was proved to guarantee the small world network property (Beaumont et al., 2007a;b).

Iterative Expanding Hashing (IEH) (Jin et al., 2014) follows the same process as KGraph to construct an approximate k-NNG; however, it refines an initial graph where the candidates for each node are generated using a hashing function. Two extensions of IEH have been proposed to better leverage advanced hashing methods for generating initial candidates: IEH-LSH (Gionis et al., 1999) and IEH-ITQ (Zhang et al., 2013). All these methods use NNDescent to finalize the graph connections.

EFANNA (Fu & Cai, 2016) selects seeds similarly to KGraph (Dong, 2022) and IEH (Jin et al., 2014) and refines candidates using NNdescent. It builds an approximate k-NNG by selecting initial neighbors of each node using randomized truncated K-D Trees (Dasgupta & Freund, 2008) and refining the graph using NNDescent (Dong et al., 2011). During search, EFANNA uses the pre-built trees to select seeds, then runs a beam search on the graph index.

Hierarchical Navigable Small World (HNSW) (Malkov & Yashunin, 2020) improves the scalability of NSW (Ponomarenko et al., 2011; Malkov et al., 2014) by proposing RND to sparsify the graph and a hierarchical seed selection strategy (SN) to shorten the search path during index building and query answering. Each hierarchical layer includes all nodes in the layer above it, with the bottom (a.k.a. *base*) layer containing all points of the dataset S, HNSW builds an NSW graph incrementally. However, HNSW diverges from NSW in that it refines the candidate nearest neighbors, identified through beam search on the nodes already in that layer using RND. During query answering, HNSW utilizes SN to quickly find an entry point in the base layer to start the beam search.

Diversified Proximity Graph (DPG) (Li et al., 2019) extends KGraph (Dong, 2022) by diversifying the neighborhoods of its nodes through edge orientation, a technique we refer to as Maximum-Oriented Neighborhood Diversification (MOND) in Section 3.4. MOND's main objective is to maximize the angles between neighboring nodes, contributing to a sparsed graph structure. This process is iteratively applied to all nodes. After that, the directed graph is transformed into an undirected one, enhancing its connectivity. Nevertheless, note that DPG's publicly available implementation (DBAIWangGroup, 2023) utilizes RND rather than MOND for neighborhood diversification.

NGT (Yahoo Japan Corporation, 2022) is an approximate nearest neighbor (ANN) search library developed by Yahoo Japan. It offers two construction methods: one extends KNN graphs with reverse edges, forming bi-directed KNN graphs (Iwasaki, 2016), while the other incrementally builds graphs similar to HNSW with a range-based search strategy (Sugawara et al., 2016). In this study, we consider the former (Iwasaki, 2016). Additionally, the library includes methods that employ quantization for highly efficient search. NGT maintains efficiency by pruning neighbors via RND and using Vantage-Point Trees (Yianilos, 1993) to select seed nodes for accurate query results.

Navigating Spreading-out Graph (NSG) (Fu et al., 2019), similarly to DPG, builds an approximate k-NNG first. But, unlike DPG, it builds an EFANNA graph rather than a KGraph. It then diversifies the graph using RND. At the end, NSG creates a depth-first search tree to verify the connectivity of the graph. If there is a vertex that is disconnected from the tree, NSG connects it to the nearest node in the tree to ensure graph connectivity.

SPTAG (Chen et al., 2018) is a library for approximate vector search proposed by Microsoft. SPTAG follows a DC approach and is based on multiple existing works. It selects small dataset samples on which it builds either K-D Trees (Beis & Lowe, 1997) or Balanced K-means Trees (Malinen & Fränti, 2014). These strutures will be used for seed selection during query answering. Then it clusters the full

dataset using multiple hierarchical random divisions of TP Trees (Wang et al., 2013), builds an exact k-NN graph on each cluster (i.e., leaf) and refines each graph using ND. The graphs are merged into one large graph index for query processing.

Vamana (Subramanya et al., 2019) is similar to NSG in considering the set of visited nodes when building long-range edges within the graph. However, instead of using EFANNA (Fu & Cai, 2016), Vamana uses a randomly generated graph with node degree $\geq\ log\,(n)$ to ensure the initial graph connectivity (Engel et al., 2004). Then, for each node, Vamana runs a beam search on the graph structure to get the visited node list R, which will be refined in the first round using RRND. After adding bi-directional edges to selected neighbors, the neighbors that exceed the maximum allowed out-degree will refine their neighborhood list following an RND process. Then, Vamana repeats the same refinement process a second time to improve the graph quality, this time using RRND with $\alpha \geq 1$ to increase the connectivity within the graph.

SSG (Fu et al., 2021) integrates the MOND approach from DPG (Li et al., 2019) and closely follows the steps of NSG (Fu et al., 2019) and DPG (Li et al., 2019) in index building from a foundational graph. Instead of performing a search for each node to acquire candidates, SSG (Fu et al., 2021) employs a breadth-first search on each node to assemble candidate neighbors through local expansion on a base graph (EFANNA). When the maximum size for the candidate neighbors is achieved, SSG reduces the neighbors in the list by enforcing the MOND diversification strategy, pruning the candidate nodes forming an angle smaller than a user-defined parameter θ with the already existing neighbors of the concerned node. After iteratively applying this method to all nodes, SSG (Fu et al., 2021) enhances connectivity by constructing multiple DFS trees from various random points, in contrast to NSG's (Fu et al., 2019) singular DFS approach.

Hierarchical Clustering-based Nearest Neighbor Graph (HCNNG) (Munoz et al., 2019) was inspired by SPTAG. It employs hierarchical clustering to randomly divide the dataset into multiple subsets. This subdivision process is executed several times, resulting in a collection of intersecting subsets. On each subset, HCNNG constructs a Minimum Spanning Tree (MST) graph. Following this, the vertices and edges from all the MSTs are merged to form a single, connected graph. To facilitate the search process, HCNNG constructs multiple K-D Trees (Beis & Lowe, 1997), to identifying entry points during query search.

HVS (Lu et al., 2021) extends HNSW's base layer by refining the construction of hierarchical layers. Instead of random selection, nodes are assigned to layers based on local density to better capture data distribution. Each layer forms a Voronoi diagram and uses multi-level quantization, increasing dimensionality by a factor of 2 in each lower layer. Search at the base layer is similar to that of HNSW.

LSHAPG (Zhao et al., 2023) combines HNSW graphs with multiple hash tables based on the LSB-Tree structure (Tao et al., 2009) to enhance search efficiency. It leverages L hash tables to retrieve seeds for beam search on the base layer, unlike HNSW, which selects a single seed through SN. LSHAPG also utilizes these hash tables for probabilistic rooting during search, pruning neighbors based on the projected distance before evaluating and pruning the raw vectors.

ELPIS (Azizi et al., 2023) is a DC-based approach that splits the dataset into subsets using the Hercules EAPCA tree (Echihabi et al., 2022), where each leaf corresponds to a different subset, then builds in parallel a graph-based index for each leaf using HNSW (Malkov & Yashunin, 2020). During search, ELPIS first selects heuristically an initial leaf and executes a beam search on its respective graph. The retrieved set of answers feed the search priority queues for the other leaves. Only a subset of leaves is selected based on the answers and the lower-bounding distances of the query to the EAPCA summarization of each leaf. Then, ELPIS initiates multiple concurrent beam searches on the graph structures of the candidate leaves. Finally, ELPIS aggregates all results from candidate clusters and returns the top-k answers.

B. Experimental Results